

Solution to Problem 20) Any function that maps the interval $(0,1)$ to $(-\infty, \infty)$, or vice-versa, can be used to establish the one-to-one correspondence between all real numbers, on the one hand, and the real numbers confined to the $(0,1)$ interval, on the other hand. The function $f(x) = \frac{1}{2}[1 + \tanh(x)]$ is one such function. Another appropriate function is $g(x) = \tan[\pi(x + \frac{1}{2})]$. The function $h(x) = \ln[\ln(1/x)]$ also maps the interval $(0,1)$ onto the entire real line between $-\infty$ and ∞ . Many other examples can be constructed along the same lines. A geometric construction that maps, via a circle of radius $\frac{1}{2}$, every point in the interval $(0,1)$ on the y -axis to the entire set of real numbers on the x -axis is shown in the figure on the right.

